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CONCEPTUALIZING INEQUALITY AND RISK

BY

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I. INTRODUCTION

Economists and finance researchers have long recognized the "close relation between the measurement of inequality and the measurement of risk" (Breitmeyer et al., 2004). Economists today agree that a measure of income inequality must respect the Pigou-Dalton transfer principle, that is, inequality cannot increase with a transfer from a richer person to a poorer person. Finance researchers today agree that a measure of risk must respect the diversification principle, that is, risk cannot increase when portfolios are combined. Where did these convictions originate? While both principles were advocated early in their respective fields, they were not viewed as definitive until relatively recently. In both fields major empirical and theoretical efforts were mounted in support of various conceptualizations at odds with these two principles. For example, Robert Gibrat advocated the use of the variance of log income despite the fact it is inconsistent with the Pigou-Dalton principle. In finance, value at risk (VaR), although it can violate the diversification principle, was widely advocated. People supporting research programs at odds with Pigou-Dalton or the diversification principle, based those programs on ambitious empirical and theoretical claims about distributions. Such claims could have been right and are not *a priori* or logically false. The purpose of this paper is to review these "at odds" conceptualizations, the types of arguments advanced for their legitimacy, and the reasons they have been given up.

Risk and inequality measure the spread of relevant distributions. Financial or economic theories that would entail a specific distribution of asset returns or incomes necessitate particular measures of risk and inequality. We will describe such theories as *structural*. For example, if all real world distributions of asset returns follow the normal distribution then risk *is* the standard deviation. If incomes follow Gibrat's Law then inequality *is* the variance of log-income. Under these circumstances, the specific structural theory dictates the measure of risk or inequality and makes redundant further conceptualization. Where structural theories are well supported by empirical measurements of the distributions involved and of their

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generative evolution, they necessarily narrow the range of possible alterations of a distribution. As is well known only a restricted set of mean preserving income transfers preserves log-normality. Hence, in a world that is "really" log-normal, many redistributive programs are impossible.[1]

The physical sciences have repeatedly demonstrated the usefulness of structural theories. Peter Caws (1957, 1959) has well described the scientific process that refines definitions of intuitive concepts as conventional wisdom gives ground to precise thinking. In the physical sciences formalization has often been part and parcel of fundamental breakthroughs in understanding and modeling the structural possibilities of the world. This structural ordering of the real world is witnessed by the ability of the sciences to harness the superficially unruly realizations of random variables; consider, for example, the statistical regularities of Brownian motion. When well-defined laws are discovered, they dictate specific measurements. Little wonder that social scientists confronted with mountains of disorganized data would seek discoveries in a similar fashion. The successes of statistical astronomy and statistical mechanics set the stage for much social scientific empiricism in the nineteenth and twentieth centuries.[2]

Economists have been eager to reproduce in their own domain the powerful intellectual simplifications that would follow from realistic structural theory. More specifically with respect to the concept of inequality, early research agendas took a strongly structural form in which inequality would be identified with the second moment of an income distribution *law*. To their credit, researchers such as Vilfredo Pareto and Robert Gibrat discovered a number of stylized facts and empirical regularities applying to observed real world income distributions. Both of these path-breaking economists embraced the structural implications of their researches in much the same spirit as the physicists of the nineteenth century. However, empirical counter-examples early on raised doubts about the "reality" of the structural formulations. This is not to say that Pareto's law or Gibrat's law weren't useful for a number of practical purposes. They were useful, approximate instrumental theories which failed to live up to the structural promise. Under these circumstances, the conceptualization of inequality remained open and eventually dictated a return to the transfer principle of Pigou-Dalton.

Where economists working on income inequality have sought to discover structure, researchers in finance have used return distributions for more pragmatic and instrumental purposes. Rather than declaring a "true structure" they have more often claimed a distribution as a workable approximation. But such a research agenda is itself subject to difficulty. Plausible approximations often prove to be laden with more structural constraints than first appreciated. For example, the second moment was first described as a useful approximation of risk by Harry Markowitz (1953). He explicitly acknowledged that the normal distribution was not a fundamental structural reality, but that the standard deviation could nevertheless do most of the heavy lifting required of a measure of risk. But this claim was confronted early on by skewed and often thick tails of asset returns. Far from an all-purpose measure, the

[1] As discussed below, this approach to "structural theories" is very much in the spirit of Chipman (1974).

[2] Historians of the new finance point out that Bachelier's now-famous treatment of the random walk nature of stock prices actually preceded Einstein's treatment of random walks (Courtault *et al.*, 2000).

standard deviation seemed perverse in a world ruled by such empirical data. Hence, a range of other statistics have been proposed as the measure of risk, value-at-risk being the most prominent recent example. These alternative measures were accompanied by a renewed consideration of the nature of risk that eventually led back to the centrality of diversification.

In section 2 we review the meanings of inequality and risk as they evolved alongside empirical statistical observations originating in the nineteenth century. We go on in section 3 to consider generative theories that sought to elaborate mechanisms that would produce specific structural distributions of incomes or returns. In such generative theories distributions arise from underlying processes that are themselves random. Unlike structural theories in the physical sciences, however, none of these structural explanations of income inequality or risk has proven sufficiently compelling as to require a single concept in its field. In section 4 we present a brief digression on the role of subjective preferences and social welfare functions as the way to define "spread concepts" absent specific structural distributions. We summarize in section 5, emphasizing the convergence between economics and finance. The failure to identify structural constraints on income distributions and the inadequacy of simple statistics to measure risk have driven both back to basic intuitions: Dalton's approach to characterize inequality via a transfer principle and the more recent but similar approach in which risk, however it is measured, is defined in terms of the diversification principle.

II. INEQUALITY, RISK AND EMPIRICAL STATISTICAL LAWS

The nineteenth century flowering of social tabulations has been well documented by historians of statistics (Hacking 1990). The practical public finance of taxation and annuities created a real need for collecting and distributing such statistics. The greater availability of tabular data perhaps inevitably generated a broader intellectual enterprise.

The early champion of social statistical laws, Adolphe Jacques Quetelet (1842), projected nothing less than a social physics. Quetelet's chief candidate for distributional laws was the normal curve used by Pierre-Simon Laplace and Carl Friedrich Gauss.[3] The search for such normal statistical laws became an obsession of the last half of the nineteenth century. Against this background, late nineteenth century work on income inequality, perhaps inevitably, became part and parcel of the hunt for distributional laws. Attempts to formalize the concept of income inequality focused on measuring the second moment of the income distribution. Inequality became a structural fact, a parameter of the normal distribution.

Much early research focused on "discovering" roots that would justify a normal law of income. Confidence in this search continued into the first decade of the twentieth century. For example, Henry Moore (1907) explicitly appealed to Quetelet's authority,

[3] While Quetelet can be credited with inventing the "average man," he didn't refer to the "law of errors" as the "normal" curve. Warren Pearson aggressively pushed the "normality" of Gauss's distribution, reflecting earlier usage by Francis Galton and Wilhelm Lexis. (We are grateful to an anonymous referee for underscoring this point.)

as well as that of Galton and Pearson, in his use of the mean and the coefficient of variation for describing wage distributions. Warren Persons (1909) also strongly endorsed the traditional measure. A. C. Pigou in *Wealth and Welfare* (1912) used the standard deviation in various theoretical arguments.

But from the start, there was an obvious problem with the structural normal model, namely income distributions were skewed, so that incomes could not be normal. If distributions were not normal, inequality would not necessarily correspond to the standard deviation. As early as 1879, Francis Galton (1879) had suggested the inadequacy of the normal curve as a description of many social and biological distributions, advocating instead the log-normal distribution. Similar concerns also motivated J. C. Kapteyn (1904), and F. Y. Edgeworth (1924), culminating in the work of Gibrat (1931) discussed in the next section.

Of the early income distribution researchers, Pareto was perhaps most self-conscious of the methodological parallels between political economy and the physical sciences. Pareto was a keen proponent of the need to move from intuitions to formal constructs. A "new-born science . . . may find it advantageous to have recourse to the common fund of experience more or less vaguely represented by words." As it matures and acquires its own "direct experience," a science must shed "the disadvantages attached to the vagueness of experience, such as is given us by everyday words." Pareto asserted, "This is precisely the state of affairs in political economy" (1897b, p. 496).

Little wonder that Pareto sought to free "inequality" from its "vagueness" and find a definition in an alternative structural form for the distribution of income. Where Galton and his followers searched for a probabilistic interpretation of skewness,[4] Pareto went so far as to deny that income inequality was generated by chance, which he interpreted as requiring the symmetric shape of the normal curve (Pareto 1897a, p. 315). Pareto instead put forth his own law of income distribution:

$$\log N = A - \alpha \log x,$$

where N is the number of households with income greater than x, A is a constant and α is a key parameter indicating the degree of inequality in the distribution. Pareto presented this law as a well-documented statistical regularity (1897a). While he acknowledged that specific exceptions might exist, Pareto took his empirical law to be a scientific foundation for a universal theory (Kirman 1987). Comparing his law to that of Johannes Kepler, Pareto looked forward to a "theory that may make this law of distribution rational in the way in which the universal law of gravitation has made Kepler's law rational" (Pareto 1897b). Inside that universal distribution for income was a measure of spread that would define inequality.

Pareto devoted considerable effort to defining inequality in the context of his statistical law, ultimately identifying inequality with the α-parameter of his distribution.[5] Since Pareto argued that α varied little across economies, he speculated that income

[4] Edgeworth (1924) called his technique for fitting non-normal curves a "method of translation."

[5] As explained by Kirman (1987), it did not help matters that a "printer's error" in Pareto (1897a) seemed to reverse Pareto's meaning.

inequality was rooted in human social psychology and largely independent of institutional reform.[6] Institutions might affect mean incomes, but not the degree of inequality (Persky 1992).

John Chipman (1974, p. 276) has made a powerful methodological case for the type of structuralist research agenda Pareto initiated: "In income-distribution theory it seems reasonable to limit attention, at least for descriptive purposes, to the types of income distribution that are likely to be observed. And if Pareto's general position is correct, it is a futile exercise to try and go very far outside of that class even for prescriptive purposes." In a footnote, Chipman elaborates on the implications for the measurement of inequality:

> fruitful discussion of such measurement requires some recognition of the existence of constraints to which income distributions may be expected to be subject, some recognition of the limitations that social, economic, and political forces place on the freedom of policy makers to manipulate income distributions at will. That such forces are as yet poorly understood should not delude us into believing that they do not exist (p. 277).

Economists early on sought empirical laws to bolster structural research agendas in income inequality. A few, like Chipman, still sense that we ignore such structure at our peril. Finance researchers have generally avoided such searches. The major claims of general statistical laws applying to asset returns originally came from the two economists most closely associated with structural arguments. First Pareto, and then Gibrat, sought to export their income distributions to finance. Thus Pareto pointed to skewed data on business rates of return, suggesting that they too followed his Pareto distribution. And Gibrat saw the lognormal distribution as an empirical match to realized distributions of corporate earnings.

While early researchers in finance acknowledged the usefulness of statistical efforts to represent historical data, they saw them as a diversion from the central problem of predicting future returns of specific companies. Despite the popular interest in various "technical" approaches to forecasting stock prices, early academic finance generally denied the relevance of historical time series for future investment. For example, see Charles Hardy (1924). The central problem in this view was predicting future returns on the basis of current company fundamentals.

While financial analysts often endorsed diversification as a wise investment strategy, their early recommendations were based on appeals to the law of large numbers. J. B. Williams (1938) had suggested that investors first pick out those instruments with the highest expected returns and then reduce risk by widely diversifying within this group. Implicit in this advice was the notion that returns across instruments were largely independent.

Markowitz motivated his famous 1952 paper on portfolio selection by criticizing Williams for not recognizing the high degree of covariance likely to exist between returns. Markowitz then went on to use covariance as the rationale for diversified portfolios. The risk associated with a portfolio composed of assets in proportions X_i, X_j,

[6] Pareto went on to make a broad argument about elites (lions and foxes) who take the same share of the pie but have a different effect on the total size.

etc. was the variance of return (V) of that portfolio:

$$V = \Sigma\Sigma\sigma_{ij}X_iX_j,$$

where (σ_{ii}) is the standard deviation of the return in asset i, and (σ_{ij}) is the covariance of the returns for i and j. Other things equal, diversification would reduce variance, namely, risk.

At the time Markowitz explicitly avoided a structural defense of his formalization of risk. Rather, he emphasized an instrumental interpretation, arguing that the variance was a useful and very likely an adequate approximation to risk. Thus he observed, "The concepts 'yield' and 'risk' appear frequently in financial writings. Usually if the term 'yield' were replaced by 'expected yield' or 'expected return' and 'risk' by 'variance of return,' little change of apparent meaning would result" (1952, p. 89).

Markowitz chose not to defend his measure of risk as rooted in empirically defined structures. Had he taken such a structural approach he would have had to argue that the distribution of asset returns followed a joint normal distribution. But he offered no such evidence.[7] His claim was that the variance as an approximation to risk was good enough to illustrate the critical relation between risk and diversification. Of course, "good enough" in this context depends on how different actual distributions are from normal. Not surprisingly then, Markowitz's claims stimulated a stream of empirical distribution fitting (Andreou, Pittis, and Spanos 2001; Poon and Granger 2003). While Markowitz had been careful in his argument and had made no explicit structural claims, his instrumental endorsement of the variance led to just the type of distribution-curve-fitting contest that had met the structural empiricism of Pareto. If Markowitz put forward a pragmatic instrumentalism, his colleagues, reasonably enough worried that plausible approximations might still run afoul of intuitions concerning the relationship between risk and diversification.

III. CONSTRUCTIVE THEORETICAL ARGUMENTS—GENERATING STRUCTURE

Any structural distribution law gains credibility when it is supported by a constructive argument as to the origins or sources of the observed distribution. In particular, when a distribution arises from simple stochastic assumptions, a measure of spread may follow naturally from the definition of that process. Historically, the reputation of the variance of the logarithms as a measure of income inequality has been particularly enhanced by such constructive arguments. Gibrat's famous law of proportional effect provided a striking demonstration of the ability of cumulative processes to generate the type of skew so common in income distributions.

[7] Indeed Markowitz acknowledged that his approach to formalizing risk differed from Williams's only if the σ_{ij}'s were sizable. But this empirical claim was not based on a new empirical law of the distribution of asset returns. Somewhat apologetically he offered no strong empirical evidence at the time.

Gibrat's (1931) *Inegalites Economiques* built squarely on the contributions of Kapteyn (1904). Edgeworth (1924) too had an inkling of the idea, while Galton had set the stage as early as 1879. But it was Kapteyn who made the clear constructive case. Significantly one of his key examples, although informal, was drawn from the world of finance:

> Suppose 10000 men to begin trading, each with the same capital; in order to seen how their wealth will be distributed after the lapse of 10 years, consider first what will be their condition at some earlier epoch, say at the end of the fifth year.
>
> We may admit that a certain trader A will then only possess a capital of 100 £, while another may possess 100,000 £.
>
> Now if a certain cause of gain or loss comes to operate, what will happen?
>
> For instance let the price of an article in which both A and B have invested their capital rise or fall. Then it will be evident that, if the gain or loss of A by 10 £, that of B will not be 10 £, but 10,000 £; that is to say the effect of this cause will not be independent of the capital, but proportional to it (p. 13).

Kapteyn went on to construct his log-normal, "skew-curve machine," a variation of Galton's famous quincunx (Figure 1).[8] Here the log-normal distribution emerges in a world that cumulates successive random draws *proportional* to a given variable. Among other applications, Kapteyn applied his technique to a distribution of housing prices under the assumption "that the main causes of gain and loss of capital, have an effect roughly proportional to the capital possessed" (p. 43). Here, Kapteyn used data that had been discussed by Pearson (p. 35). He correctly criticizes Pearson (and Quetelet before him) for assuming that skew curves are generated, "when the tendency to deviation on one side of the mean is unequal to the tendency to deviation on the other side" (quoted by Kapteyn from Pearson, vol. 186 *Philosophical Transactions*). Kapteyn points out that such a stochastic process will eventually "converge on a normal curve" no matter what the starting point. Skew originates rather in the dependence structure of the deviation.

Building on Kapteyn, Gibrat's contribution was largely to demonstrate a range of empirical applications. In his famous thesis, Gibrat offered applications from both income inequality and finance, i.e., rates of return of corporations, as well as from city and firm sizes. Gibrat related all of these distributions to his law of proportional effects.

A key problem with Gibrat's law is the fact that over time, the observed variance will tend to infinity. This bothered a number of researchers, including Michal Kalecki who suggested at least a mechanical way to generate a bound on the variance in his 1945 paper. This line of research continued into the early 1950s when several economists offered stochastic models similar in spirit to Gibrat's but varying in their choice of random processes (Champernowne 1953, Mandelbrot 1960). While these developments created a broader range of tools, their very multiplicity undermined the simple parable of Gibrat's demonstration, as well as his identification of inequality

[8] For a very nice virtual rendition of Kapteyn machine see Gut, Limpert, and Hinterberger's "Modelling the Genesis of Normal and Log-Normal Distributions" available at: http://www.inf.ethz.ch/ ~ gut/lognormal/. In addition to the virtual model, this site contains a number of links to other sites concerned with the log-normal distribution.

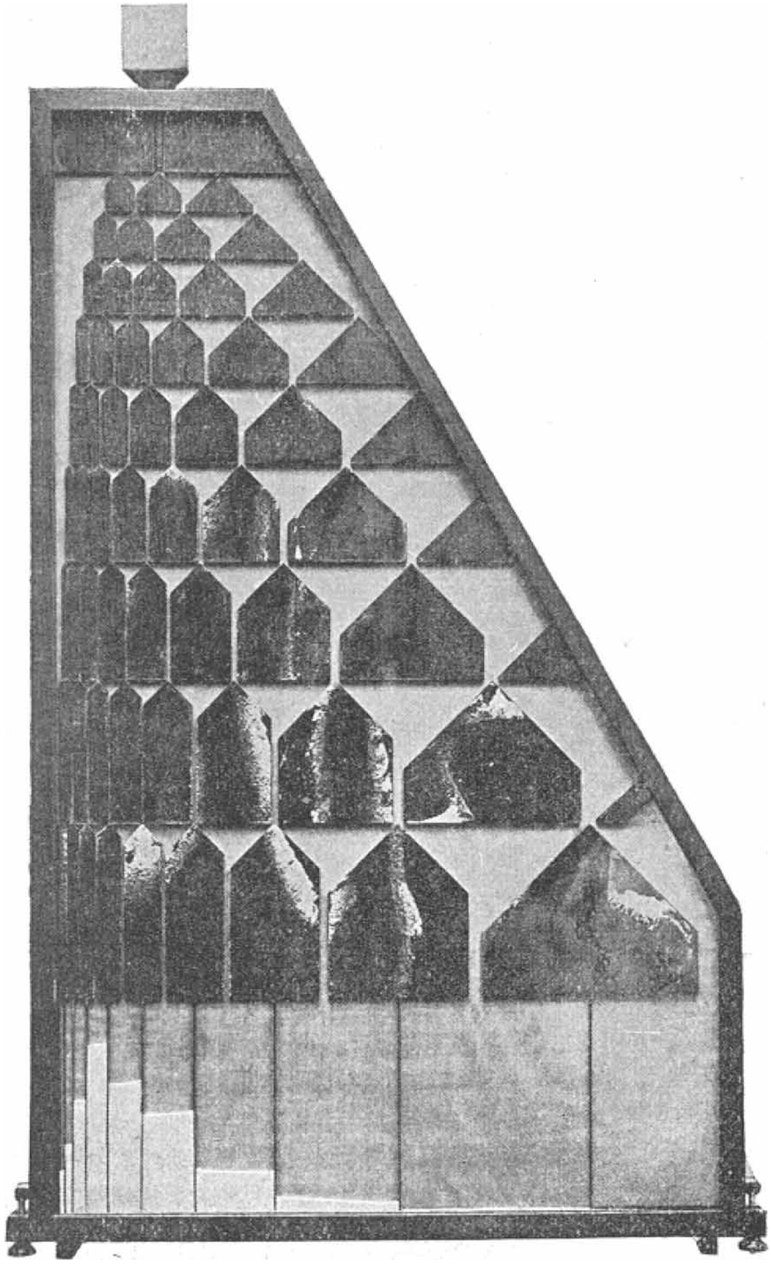

Figure 1. J. C. Kapteyn's Machine. *Source*: Reproduced from Kapteyn (1904)

with the variance of the logarithms. In all these models we recognize the presence of "inequality," yet each model offered a different operational formula for its measurement. An approach originally meant to simplify the problem of defining inequality, instead made it contingent on an increasing set of possible distributions.

Perhaps most upsetting to many economists (who wanted a non-stochastic rationale for income determination), the various definitions of inequality-through-random-process fashioned by Gibrat and others explained differences in income and wealth solely on the basis of a random process. It is one thing to use observed regularities in the income distribution to suggest definitions of inequality, quite another to anchor the definition of inequality in a purely random process. Not a few economists thought such an approach to inequality left out too much. For these researchers generative arguments based on random occurrences, though clever, did not strengthen the case for distributional structure.

Yet, if economists at mid-century found such definitions of income inequality too artificial, pragmatic finance utilized primitive random processes in descriptive theories of asset prices. Indeed they increasingly claimed the theoretical and empirical analysis of random processes as the central theme of their discipline and argued that risk had long been identified with random walks. They pointed to the work of Louis Bachelier, a student of Henri Poincare at the Sorbonne. In his now famous thesis, *Theorie de la Speculation*, Bachelier applied a normal random-walk process to short term price movements of securities (Courtault *et al.* 2000).[9]

While the findings of Bachelier went largely unheralded until the 1960s, the general approach of cumulative random processes had already struck a receptive chord in American academic finance by the 1930s. These developments seemed to have owed more to Slutsky's discussion of cycles than to Bachelier. For example, Holbrook Working (1934) cites the former, and not the latter, in a key paper emphasizing the random nature of first differences in stock prices.

But whatever the pre-1950 pedigree of random processes in finance, in the second half of the twentieth century, academic financial research in the United States had little reason to reject definitions that constructed "risk" out of purely random processes. But such general agreement hardly favored one theory over another. It proved just too easy to generate any number of plausible random processes. The multiplicity of choices left the conceptualization of risk still problematic. All the generative theories involved definitions of risk, but not all could be right.

IV. DEFINITIONS FROM UTILITY FUNCTIONS

Up to now we have restricted ourselves to discussing conceptions of inequality and risk in terms of distributions. However, from early in the twentieth century onwards, some economists sought the definition of inequality in terms of a social welfare function. In this view inequality would be defined as the difference in actual social welfare relative to the aggregate utility achievable with complete equality. Most notably, Hugh Dalton (1920, p. 349) argued that inequality was best defined as the ratio of the "total economic welfare attainable under an equal distribution to the

[9] Most remarkably, as discussed by Jovanovic and Le Gall (2001), almost forty years before Bachelier's effort, Jules Regnault, inspired by the works of Quetelet, had searched for the mean values of financial prices generated by what we now call random walks. Jovanovic and Le Gall make clear, however, that Regnault viewed the random walk of daily prices as a type of "error," his prime concern being the deterministic means beneath the surface (p. 337).

total economic welfare attained under the given distribution." As Dalton suggested, this notion of inequality builds easily on nineteenth century utilitarianism coupled with the basic neoclassical belief in diminishing marginal utility.[10]

Dalton held that we lack any certainty about the true social welfare function and hence the measure of inequality that corresponds to it. He went on to offer the transfer principle as a broad test of alternative measures of inequality. Dalton recognized that the simple sum of identical utility functions passes this test, as does the standard deviation and the Gini coefficient. But as Dalton noted, a number of others, such as the mean deviation and the inter-quartile variation do not. As to Pareto, Dalton simply recognized that the contention that any transfer would *cause* a change in mean income rules out the transfer test upon which his notion of inequality was constructed.

Interestingly, in commenting on Dalton's paper, Corrado Gini (1921) explicitly eschewed the use of social welfare functions in justifying his index. Rather he argued it was a measure of inequality, which, like the standard deviation, was applicable to a collection of measurements of any variable. For Gini, inequality had nothing to do with subjective measures of welfare.

Like inequality, risk can be defined as a property of utility functions. The basic relationship of diversification and indifference maps was first well-articulated by J. R. Hicks. Although his major interest at the time was the theory of money, he clearly enunciated the modern diversification argument.[11] Having described "frequency curves" of alternative investments, Hicks suggested "each curve could be rigidly defined by taking a sufficiently large number of moments, and an approximation to the situation obtained by taking a small number" (Marschak 1934). Subject to an understanding that "in most cases" combining risky projects will lower risks, we "expect to find our representative individual distributing his assets among relatively safe and relatively risky investments; and the distribution will be governed, once again, by the objective facts upon which he bases his estimates of risk, and his subjective preference for much or little risk-bearing"(Hicks 1935). Thus Hicks anticipates the notion of an investor's indifference map as between mean return and risk, and its usefulness in explaining diversification.

This train of thought attracted relatively little attention in finance until a similar logic was incorporated by Markowitz.[12] Markowitz drew heavily on the Friedman-Savage theory of risk.[13] Markowitz focused only on mean and standard deviation in his characterization of the distribution of returns for an asset. As discussed above, one justification of this position is to assume that returns on assets are joint normal

[10] Dalton, like Pigou, Marshall, and J. S. Mill, was well aware that transfers might have an opportunity cost in the form of reduced output. Perhaps the major advantage of the social welfare approach to measuring inequality was that it provided a sharper focus on the traditional utilitarian trade-off between overall production and equality (Persky 2002). This theme is still very central to discussions of social welfare in the work of Shorrocks (1983).

[11] The points are made by Hicks (1935) in the monetary context, with references back to his presentation at the 1933 Econometric Society meeting as reported in Marschak (1934). Marschak was Markowitz's thesis adviser (Rubinstein 2002).

[12] Markowitz cites Hicks in this paper, but only to criticize Hicks's approach to expected returns in *Value and Capital*. Markowitz doesn't seem to know of Hicks's other work although there may have been a connection through Marschak.

[13] It is worth noting that Friedman took this same starting point to suggest that much of the inequality in the distribution of income was the result of conscious choices by individuals across available lotteries (Friedman 1953).

in their distribution. As is now well known, the only other formal justification of Markowitz's decision rule assumes a quadratic utility function. However, Markowitz himself raised doubts that such utility functions apply universally. An interest in gambling and risk taking, Markowitz argued, suggests the inclusion of the third moment of a distribution into the decision maker's utility function. Without offering a specific form, Markowitz acknowledged the possibility of such utility functions.[14] Under the circumstances, Markowitz's implicitly falls back on the normality justification.

Normative influences in conceptualizing risk have continued. The fact that the variance increases due to outcomes in the right-hand tail of a distribution suggested the need for alternative measures such as the semivariance (Markowitz, 1959). These measures focus on the outcomes in the left-hand tail of the distribution and hence better match our intuitive sense of what is "riskier." Similarly, the value at risk, (the α quantile, of a distribution) focuses attention on losses exceeding a threshold value. These represented *ad hoc* modifications meant to assuage uneasiness with definitions that allowed "good" changes in a distribution to raise a measure of risk. Yet they easily ran afoul of their own very specific structural implications. More specifically, for many situations they violated the principle of diversification.

The recent history of VaR exemplifies the disillusionment that follows on attempts to introduce *ad hoc* normative notions. Giorgio Szego (2002, pp. 1260–61) highlights the irony of VaR's fall as an all purpose measure of risk:

> VaR in general turns out to be not even weakly coherent and in particular not subadditve. To try and measure risk without this property is like measuring the distance between two points using a rubber band instead of a ruler! Only in the special case in which the joint distribution of returns is elliptic is VaR subadditve . . . Thus VaR, that was introduced in the attempt of measuring risk for weird distributions, can be used only when the computationally simple variance can be used!

This passage also makes explicit the recently established consensus that the diversification principle (here referred to as subadditivity) must be the touchstone in defining risk.

V. CONCLUSIONS: NECESSARY CONDITIONS

Dalton offered his transfer principle as a test for measuring the consistency of any inequality measure with basic utilitarian ideas about social welfare. He presents a semi-formal derivation of his transfer principle from those premises. Gini and others found the argument unconvincing. They objected to a definition of inequality based on such normative evaluations. Yet Gini's measure passes Dalton's test. More generally, we can still accept Dalton's transfer principle as a basic construct in its own right. The transfer principle then can be identified as a necessary condition of any definition of inequality. Whatever inequality is, it decreases when income is

[14] Indeed, Markowitz goes so far as to suggest that an individual with a utility function with just mean and variance as arguments might be called an investor, while an individual whose utility function included higher order moments might be called a gambler.

transferred from a rich person to a poor one. While this fails to uniquely identify a single measure of inequality, it usefully formalizes our intuitions about inequality.

Our conclusions with respect to risk are surprisingly similar. At least partially in response to the explicitly normative character of modern definitions of risk, research is now based on a new axiomatic approach. The starting point for this work is the seminal paper by Philippe Artzner *et al.* (1999) introducing so called coherent risk measures. The fundamental axiom in this approach is that diversification should never increase risk, and is not unlike Dalton's requirement on income transfers. It translates into the formal requirement that risk measures, among other things, should be sub-additive.

For both inequality and risk, the effort to define concepts derived from well-articulated structural models has largely been abandoned. History suggests, but can hardly prove, that neither of these concepts will ever be narrowly defined in this structural sense. Researchers in both fields will continue to use specific formulaic measures and such efforts will add to our empirical and theoretical knowledge of inequality and risk. However, at a methodological level, the histories of these two concepts point to the usefulness of necessary conditions as primitives that may not be unique, but yet can be formalized from intuitional principles in a rigorous manner.

The history of the conceptualization of risk and inequality can perhaps be best characterized as a convergent evolution. Despite broad differences in general outlook (Granger 2001) and specific differences in their approaches to dispersion—the eagerness of economists to search for structure and the instrumentalism of finance—has resulted in similar solutions to the conceptualization of inequality and risk.

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